A Random-Sampling Procedure with Applications to Structural Synthesis Problems

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The general problem of optimization with arbitrary constraints is treated using random numbers and Monte Carlo sampling techniques. The validity of the technique, when applied to structural synthesis problems, is demonstrated by comparing the results to published results obtained by means of the gradient method. It is shown that the technique, being approximate, did not produce better design configurations than those given by the gradient method; however, they are consistently close enough to be acceptable. In another application, the results of this random-sampling technique are compared with those obtained by means of conventional methods. These results show that, in 60% of the comparisons, the random-sampling technique produced lighter weights. The indications are that this generalized and readily applicable synthesis approach will enable the designer to investigate several different design concepts for their relative design values without undue waste of time and effort. Two main advantages of this technique are that there are no restrictions on any of the constraint and merit functions, and that any number of variables and constraint conditions can be used.

Nomenclature

A = subspace of acceptable design points
a_i = acceptable design point ith approximation; also length of ith panel
b_f = stiffener flange length
b_i = width of ith panel
b_w = stiffener height
b_x = stiffener spacing
c_i = height of ith panel
F = merit function to be optimized
G = combined constraint boundary between A and U
g_i = ith constraint function
I_x = moment of inertia in x direction
I_s = moment of inertia of sheet only
L_k = number of divisions of kth dimension of s_6
m = sample size
N_s = total number of pockets in s_6
n = number of variables
P = symbol indicating probability
S = design space
s = design subspace
s^* = subspace of optimum design
t_f = stiffener flange thickness
t_s = cover sheet thickness
t_w = stiffener web thickness
U = space of unacceptable design points
V_i = index number of sections δs_6
x = point in design space, representing a design configuration
x^* = point in subspace of optimum design
ξ_i = numerical value of ith design variable

Introduction

THE objective of this study is to develop a program that will produce realistic optimum structural design configurations when the restrictions on the variables and the aims of the design are given as the input data. The conventional method consists of equating the local and general buckling loads for a given design and solving for the ratios of the design parameters. Three serious limitations of this approach make it unsuitable for a general optimization procedure: 1) In case of a multiplicity of combined loading conditions, optimum design cannot be obtained. 2) For more than one local buckling condition, the combination of equations is not possible for a single buckling load.[1] 3) No manufacturing or other arbitrary constraints can be placed on the design parameters. The search for the solution of the general structural optimization problem was confined to the methods of general optimization.[2]

Because of the nonlinearity of the constraint conditions and merit function, all methods requiring the linearity conditions were discarded. The only remaining semideterministic ones† were the direct differential gradient method and the Lagrangian differential gradient method.[3] The direct gradient method has been partially used to optimize symmetric waffle plates and the results are satisfactory.[4]

A modified gradient method was developed to increase efficiency of the direct differential method, and improved results were obtained.[5] However, its rather complicated features led to the present probabilistic approach studied in this paper.

Application of Random-Sampling Method to Structural Synthesis

Structural synthesis can be defined as the rational selection and improvement, in terms of weight or cost, of a structural design configuration without violating any of the given failure conditions and the manufacturing or design limitations. The conventional way of designing an efficient structure is by utilizing the designer's experience and judgment, and proceeding, by trial and error, until a satisfactory solution is found.

The idea of applying the random-sampling (RS) method to the solution of structural synthesis problems is similar to the course of action between an RS-type solution and the designer's way of solution previously described. A point in favor of the RS method is that it can be applied to most types of structural synthesis problems without use of much statistical theory. Finally, being a completely random procedure, it is not prejudiced unless biased by the programmer.

For illustration, assume that the structure to be designed has three variables, thickness, spacing, and height, and each

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† They are called semideterministic because the end result is dependent upon the initial starting point.

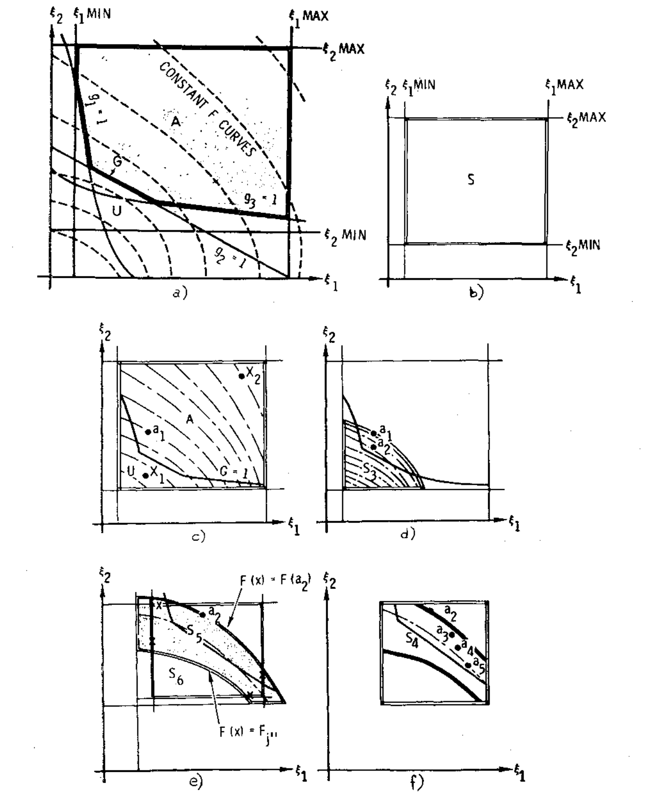


Fig. 1 Steps of searching for the zone of optimum design.

one has given limitations. This design can be expressed as $x = (\xi_1, \xi_2, \xi_3)$ where ξ_1, ξ_2, ξ_3 are thickness, spacing, and height, respectively. The restrictions are local and general stability for a given load, and the merit function is the weight. This problem can be solved by checking all the possible design configurations, i.e., all the distinct x's for local and general stability, and picking the one with the minimum weight among the stable configurations.

Assuming that thickness, spacing, and height are 100 values each, the number of distinct design configurations is 1,000,000. Since there are two stability conditions in addition to the weight, there are 3,000,000 computations if all the points have to be checked. The function of the RS procedure is to cut the number of computations to a few hundred so that it can be performed economically.

Importance sampling is used so that random samples are drawn from zones where the probability of success is high. In other words, "the biasing is done in such a way that the probability of the sample's being drawn from an interesting region is increased."[6]

Basic Screening Steps of the Program

The problem consists of locating a design point in the space of all possible design points such that all the design requirements are satisfied. The merit function evaluated at that point is as close to its optimum value as desired. Such a design point is denoted as x^*. Then initially we have

$$P(x = x^*) = P(x \epsilon S^*) \qquad \text{where} \qquad x \epsilon S \tag{1}$$

S is the n-dimensional space of design variables where n is the number of variables for the particular problem. The position of each design point x in this space is specified by the values of its coordinates ξ_k:

$$x = (\xi_1, \xi_2, \ldots, \xi_n) \tag{2}$$

The boundaries of the design space are specified by the minimum and maximum allowable values of the design variables. These boundaries are commonly referred to as side constraints in the literature.

The size of S^* depends on the desired accuracy of the solution. Assuming the optimum point x^o has the coordinates

$$x^o = (\xi_1{}^o, \xi_2{}^o, \ldots, \xi_n{}^o) \tag{3}$$

if we can accept another design point x^* in the neighborhood of x^o such that

$$|\xi_i{}^o - \xi_i{}^*| \leqslant \eta_i \qquad \text{where} \qquad \eta_i \geqslant 0, \quad i = 1, 2, \ldots, n \tag{4}$$

then an S^* can be defined as the subspace of all x^*'s. In design problems, the number of significant digits is limited for practical reasons; therefore, the random variable ξ_i can only take discrete values. If the number of significant digits is d_i, we have for the density function $f(\xi_i)$

$$\begin{aligned} \xi_i < \xi_i \text{ min}: &\quad f(\xi_i) = 0 \\ \xi_i \text{ min} \leqslant \xi_i \leqslant \xi_i \text{ max}: &\quad f(\xi_i) = \frac{10^{-d_i}}{(\xi_i \text{ max} - \xi_i \text{ min})} \\ \xi_i > \xi_i \text{ max}: &\quad f(\xi_i) = 0 \end{aligned} \tag{5}$$

For the case of integer variable, the corresponding d_i is taken equal to zero.

Since ξ_i is uniformly distributed between its maximum and minimum values, the probability of $|\xi_i{}^o - \xi_i| \leqslant \eta_i$ is given as

$$P(|\xi_i{}^o - \xi_i| \leqslant \eta_i) = 2\eta_i/(\xi_i \text{ max} - \xi_i \text{ min}) \tag{6}$$

and the probability of a randomly chosen x being in S^* is therefore

$$P(x \epsilon S^*) = \prod_{i=1}^{i=n} \frac{2\eta_i}{(\xi_i \text{ max} - \xi_i \text{ min})} \tag{7}$$

which is expression (1) written in a different form.

The space S is defined in such a way that every point in it satisfies the side constraints; however, there are other constraints which have to be satisfied in order to meet the design requirements, like the various strength conditions in structural synthesis problems, and these are referred to as main constraints. If there are k main constraints and n variables, there are $k + 2n$ constraints including the side constraints, and we have

$$g_i(x) < 1 \qquad \text{for all } x \epsilon S$$
$$i = k + 1, k + 2, \ldots, k + 2n$$
$$g_i(x) < 1 \qquad \text{for all } x \epsilon A \qquad i = 1, 2, \ldots, k + 2n$$
$$g_i(x) \geqslant 1 \qquad \text{for all } x \epsilon U \qquad i = 1, 2, \ldots, k$$

where A and U are the subspaces of acceptable design points and unacceptable design points, respectively. They have the following properties:

$$U \cup A = S \qquad U \cap A = \phi \tag{8}$$

where ϕ is the empty space.

A design point is considered unacceptable if it violates any of the constraint conditions. The only requirement for the g_i is that it must have a computable value for every x in S.

The concepts mentioned so far have been illustrated in Fig. 1a for a problem with only two variables. The boundary between U, and A is designated by G and, in the general case, G represents a hypersurface with concave and convex portions.

The program operates by picking a random point and then checking it against the given constraints to determine whether it is in A or U, and continues until it cannot find a point in A with a merit function lower than the previous one. (See Fig. 2.)

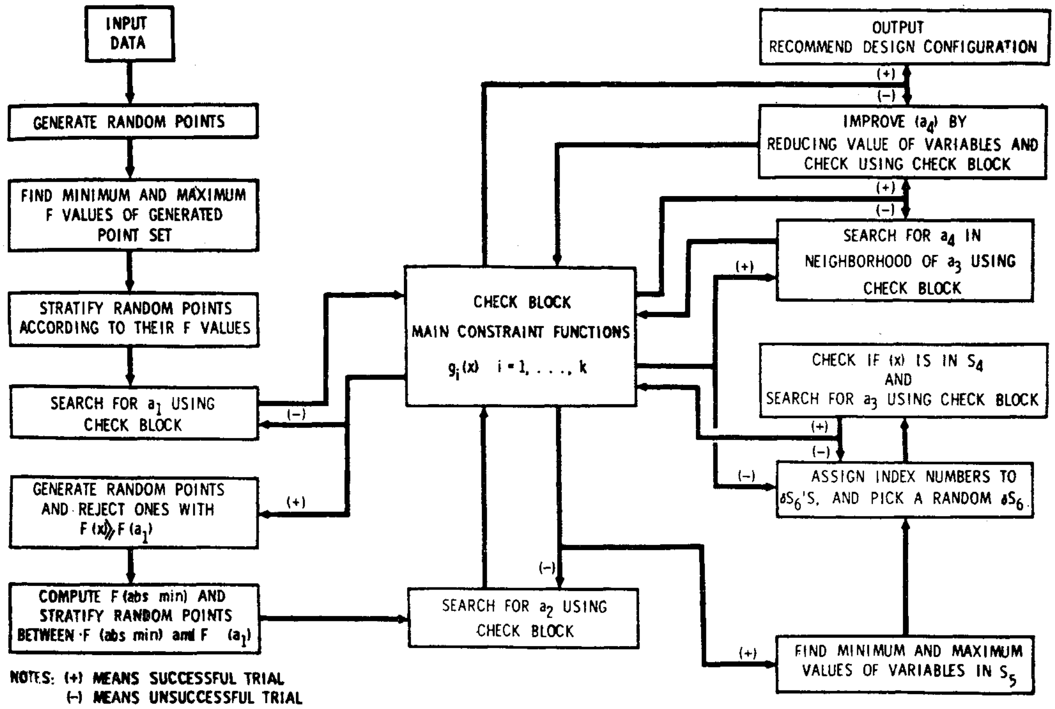


Fig. 2 Flow diagram for computational procedure.

The merit function $F(x)$ is the function to be optimized. It has a unique value for every x, and it is computable. In order to improve the probability of success with a minimum number of computations, a system of sampling techniques is utilized which is described in later paragraphs.

The operation is based largely on the assertion that the evaluation of main constraints for a given x demands an effort much larger than the computation of merit function for that design point; therefore, it must be avoided as much as possible, and the information obtained from the merit function values must be fully used.

The procedure is initiated by picking a random sample of size m from S. If it is possible to make at least an estimate on the probable relative size of A with respect to S, say p %, m can be computed using Poisson's approximation P (number of design points found in $A \geqslant 1$) $= 1 - e^{-m_1 p}$ and, if m_1 is such that

$$m_1 \geqslant 4/p \tag{9}$$

the probability of finding at least one acceptable point among the points initially sampled is greater than 0.982.

The rest of the procedure consists of the following steps:

1) Calling the subspace induced by the m_1 design points initially picked s_1, $F(x)$ is evaluated for every x in s_1 and the highest and lowest values of $F(x)$ attained in s_1 are denoted as F_1 max and F_1 min, respectively.

2) The subspace s_1 is sliced into $\dot{J}$ sections such that a design point in the jth slice has a merit function value between F_j and F_{j-1}, where

$$F_j = (j/\dot{J})(F_1 \max - F_1 \min) + F_1 \min \tag{10}$$

Every point in s_1 is then denoted as

$$x(i;j) \qquad i = 1,2, \ldots, m_1 \qquad j = 1,2, \ldots, J \tag{11}$$

according to the order of picking and its merit function value. It is reasonable to assume that, as the value of j increases, the probability of $x(i;j)$ being in A increases. This can be seen without difficulty if the G is concave and F is convex. (See Fig. 1c.)

3) The probability density function for the initial sample as a function of j is computed as $f(j)$ (see Figs. 3 and 4):

$$f(j) = \text{number of points in each slice}/m_1 \tag{12}$$

4) Among the points in the first slice, one is picked at random: $x(l; 1)$, and $g_i[x(l; 1)]$ is computed for every i, $i = 1, 2 \ldots, k$. If

$$g_i[x(l; 1)] < 1 \qquad \text{for} \qquad i = 1, 2, \ldots k \tag{13}$$

it is concluded that $x(l; 1)$ is in A and called a_1; otherwise, a new point in the first slice is selected for testing. After the number of points checked reaches a value equal to l_j, and a value a_1 still has not been found, the first slice is abandoned and a point from the next slice is selected. Here

$$l_j = m_1 f(j)/j \tag{14}$$

The reason for this is that it is more desirable to find a design point in slices with low j values because they will be closer

a) **Z-stiffened cover plate**

b) **Symmetric waffle plate**

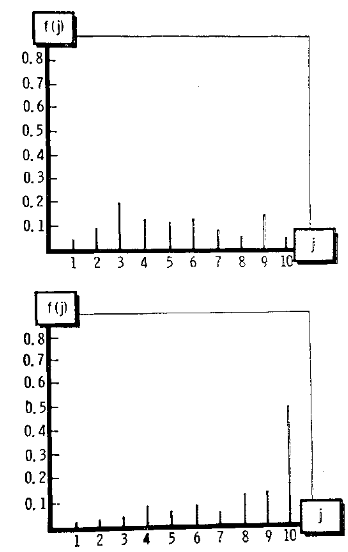
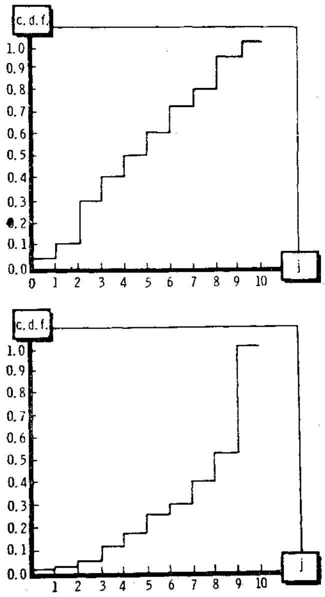


Fig. 3 Density of design points according to their merit values.



a) *Z*-stiffened cover plate

b) Symmetric waffle plate

Fig. 4 Cumulative distribution of design points according to their merit values.

to S^*. This procedure is continued till a_1 is found, a_1 being the first design point found in A.

5) Assuming that the probability distribution of $F(x)$ computed for s_1 applies to S as well, $P_1 = P[F(x) < F(a_1)]$ computed for an x picked at random in S.

6) A new sample is picked from S with a sample size m_2, where

$$m_2 = m_1/P_1 \tag{15}$$

The subspace induced by m_2 design points is called s_2, and $F(x)$ for every x in s_2 is computed. After eliminating all the design points with an $F(x)$ higher than $F(a_1)$, a new subspace s_3 is defined consisting of m_3 points such that

$$F(\text{abs min}) \leqslant F(x) \leqslant F(a_1) \qquad \text{if} \qquad x \epsilon s_3 \tag{16}$$

where F(abs min) is the merit value of the design point that has the minimum design variable values as its coordinates. Minimum and maximum values of the design variables were given as the side constraints.

7) In this step, the subspace s_3 is sliced into J sections and an index value j' is assigned to each point in s_3 such that

$$F_{j'} = (j'/J)[F(a_1) - F(\text{abs min})] + F(\text{abs min}) \tag{17}$$

8) Proceeding in a similar fashion to the procedure of step 4, a new acceptable point a_2 is found in s_3 with a merit function value lower than $F(a_1)$.

9) So far, several points have been checked against the design requirements and two points, a_1 and a_2, have been located in A. Assume the number of points checked in s_3 before the a_2 is found is m_4. It is also known that they have lower merit function values than $F(a_2)$. If we define

$$P_2 = P(x \epsilon A / x \epsilon s_3) \tag{18}$$

the probability of not finding an acceptable point among the m_4 points, P_o, can be computed using the Poisson's approximation:

$$P_2 = (-\log P_o/m_4) \tag{19}$$

Assuming a reasonable value for P_o, say 95%, P_2 can be computed and it can be stated that there is a 95% probability that P_2 is equal to, or less than, its value computed by expression (19). Since we also know the number of points listed in s_3, the probable number of acceptable points among m_3 is given as m_3P_2. If m_3P_2 is less than or equal to 2, the operation is stopped and a_2 is considered a good approximation to x^o, otherwise, a new space s_5 is defined as the space of all x's in s_3 such that

$$P_{j''} \leqslant F(x) < F(a_2) \tag{20}$$

where j'' is the nearest integer value to $0.5 + (1 - P_2)j'(a_2)$, and $j'(a_2)$ is the j' value corresponding to a_2.

10) By examining the coordinates of all the points with a merit function value less than $F(a_2)$, it is possible to replace the original side constraints by means of fictitious ones based on the highest and lowest values of the design variables observed. (See Fig. 1e.) In other words, s_6 is a subspace of S bounded by the minimum and maximum values of the design variables found in s_5.

11) In this step, s_6 is divided into pockets, δs_6, and a portion of these pockets is investigated. This is done by dividing the distance between the boundaries into L_k equal portions, with each δs_6 specified by an index number such as $(V_1;\ V_2;\ \ldots;\ V_n)$. The minimum and maximum values of design variables for each pocket are given by

$$\xi_k \min(V_1;\ V_2;\ \ldots;\ V_n) = [\xi_k \max(s_6) - \xi_k \min(s_6)] \times (V_k - 1)/L_k + \xi_k \min(s_6) \tag{21}$$

$$\xi_k \max(V_1;\ V_2;\ \ldots;\ V_n) = [\xi_k \max(s_6) - \xi_k \min(s_6)] \times V_k/L_k + \xi_k \min(s_6) \tag{22}$$

Total number of δs_6 is N_s where

$$N_s = \prod_{k=1}^{k=n} L_k \tag{23}$$

12) Using random numbers, a pocket is selected, a design point $x(V_1;\ V_2;\ \ldots;\ V_n)$ corresponding to the center of that pocket is found, and its merit function value $F(V_1;\ V_2;\ \ldots;\ V_n)$ is computed. If

$$F_{j''} \leqslant F(V_1;\ V_2;\ \ldots;\ V_n) \leqslant F(a_2) \tag{24}$$

then

$$x(V_1;\ V_2;\ \ldots;\ V_n) \epsilon s_5 \tag{25}$$

Therefore, it is from an interesting zone, called s_4, where

$$s_4 = s_5 \cap s_6 \tag{26}$$

13) Design points coming from interesting zones described in step 12 are checked against the constraint boundary G until one in A is located, and denoted as a_3.

14) After a_3 has been established, the pockets neighboring the one containing the a_3 are checked by means of keeping all the values of V_k constant except one, in the index corresponding to a_3.

15) If this search establishes a new design point in A with a lower merit function value, the process is continued; otherwise, a_3 is accepted as the recommended design point.

Explanation and Discussion of Results

It can be stated that the end result of this study is the establishment of a synthesis procedure in the form of a computer program. The validity of the random-sampling tech-

Table 1 Comparison of random-sampling (RS) results with Schmit and Kicher (S & K) results

	Case	Weight, lb	t_s, in.	b_x, in.	t_w, in.	% extra weight
S & K	3-1	139.6	0.394	3.182	0.011	...
RS	3-1	147.2	0.400	2.860	0.249	5.5
S & K	3-2	187.2	0.052	3.705	0.692	...
RS	3-2	188.4	0.062	3.863	0.639	0.6
S & K	3-3	193.96	0.395	4.072	0.010	...
RS	3-3	200.79	0.109	5.893	2.397	3.7

Table 2 Results of structural synthesis for wing cover panel with Z stiffeners

Panel	Run	a, in.	b, in.	c, in.	$-N_x$, in.	$\bar{t}$, in.	b_x, in.	t_s, in.	b_w, in.	t_w, in.	W, lb	$\frac{(W - W_o)100}{W_o}$	G_1	G_2	G_3	G_4
1	0[a]	18.0	69.0	1.5	9.40	0.188	2.89	0.111	1.450	0.080	23.800	...	0.806	0.917	0.905	0.254
1	1	18.0	69.0	1.5	9.40	0.172	2.89	0.141	1.019	0.057	21.758	−8.5	0.826	0.590	0.926	0.928
1	2	18.0	69.0	1.5	9.40	0.178	2.89	0.144	1.025	0.061	22.443	−5.6	0.800	0.547	0.825	0.891
1	3	18.0	69.0	1.5	9.40	0.181	2.89	0.138	1.098	0.076	23.150	−3.0	0.776	0.576	0.738	0.672
1	4	18.0	69.0	1.5	9.40	0.178	2.89	0.134	1.097	0.072	22.416	−5.6	0.801	0.628	0.789	0.686
1	5	18.0	69.0	1.5	9.40	0.178	2.89	0.107	1.072	0.117	22.456	−5.6	0.800	0.953	0.539	0.701
1	6	18.0	69.0	1.5	9.40	0.178	2.89	0.104	1.031	0.131	22.731	−4.5	0.790	0.955	0.474	0.787
2	0	20.0	64.0	1.5	8.20	0.169	2.40	0.087	1.450	0.080	21.600	...	0.746	0.940	0.836	0.238
2	1	20.0	64.0	1.5	8.20	0.178	2.40	0.103	1.017	0.105	22.798	+5.5	0.708	0.627	0.498	0.798
2	2	20.0	64.0	1.5	8.20	0.181	2.40	0.109	1.337	0.077	23.214	+12.0	0.696	0.564	0.757	0.313
3	0	21.0	49.0	1.5	8.30	0.171	1.90	0.067	1.450	0.080	17.600	...	0 748	0.978	0.836	0.222
3	1	21.0	49.0	1.5	8.30	0.167	1.90	0.118	1.028	0.054	17.174	−2.5	0.765	0.334	0.896	0.760
3	2	21.0	49.0	1.5	8.30	0.190	1.90	0 079	1.131	0.110	19.625	+11.5	0.700	0.608	0.496	0.531
4	0	15.0	41.5	1.5	8.10	0.156	1.75	0.066	1.150	0.080	9.700	...	0.802	0.911	0.756	0.244
4	1	15.0	41.5	1.5	8.10	0.169	1.75	0.071	1.182	0.086	10.538	+8.5	0.736	0.724	0.672	0.218
4	2	15.0	41.5	1.5	8.10	0.152	1.75	0.097	0.977	0.058	9.449	−2.5	0.820	0.445	0.872	0.429
4	3	15.0	41.5	1.5	8.10	0.165	1.75	0.080	0.853	0.103	10.284	+6.0	0.754	0.566	0.473	0.661
4	4	15.0	41.5	1.5	8.10	0.179	1.75	0.134	0.969	0.048	11.129	+15.0	0.697	0.200	0.849	0.434
5	0	18.0	36.0	1.5	5.56	0.129	1.75	0.062	1.150	0.060	8.360	...	0.663	0.874	0.780	0.247
5	1	18.0	36.0	1.5	5.56	0.127	1.75	0.096	0.849	0.037	8.215	−1.9	0.675	0.380	0.900	0.756
5	2	18.0	36.0	1.5	5.56	0.130	1.75	0.099	0.836	0.038	8.415	+0.8	0.659	0.347	0.862	0.795
6	0	20.0	31.0	1.5	3.60	0.109	1.75	0.053	1.150	0.050	6.760	...	0.508	0.928	0.684	0.202
6	1	20.0	31.0	1.5	3.60	0.970	1.75	0.062	0.910	0.040	6.023	−10.9	0.570	0.774	0.758	0.487
6	2	20.0	31.0	1.5	3.60	0.133	1.75	0.095	1.361	0.029	8.254	+22.0	0.416	0.243	0.955	0.104

[a] Run 0 indicates hand-computed values by designer using conventional methods. It is included for comparison.

nique applied to the structural synthesis problem is demonstrated by comparing the results of this program to Schmit and Kicher results.[4] In Table 1, column 7, the relative percentage values are given. As previously mentioned, the random-sampling method, being an approximate method, did not produce better design configuration values than the ones given by the gradient method. However, it consistently produced results close enough to be acceptable during the preliminary design stage.

The random-sampling technique was also applied to a wing cover panel, preliminary-design problem and the results were checked against the values computed by the designer using conventional methods. The results are listed in Table 2.

The interesting part of the comparison between the values obtained by conventional methods and the results of random sampling is that 60%, of the runs produced weight values lighter than the comparative design weight (see Table 2, column 13). This was because the designer's value, unlike the values used for comparison in Table 1, was not the absolute minimum.

Constraint values were tabulated in Table 2, columns 14-17, and they describe the position of the recommended design configuration with respect to main constraint boundaries. The value "one" indicates a design configuration with a zero margin of safety for the relevant constraint condition. For example, Table 2, panel 1, run 6, a value 0.995 for G_2 indicates that this particular design has almost zero margin of safety for the local buckling of the cover sheet. Furthermore, the value of G_3 (0.474) in the same design indicates that the margin of safety for the local buckling of the stiffener is high; therefore, a lighter stiffener can be tried for the final design.

Conclusions

It is demonstrated that the random-sampling technique can be utilized for the everyday preliminary-design problems even in its present crude form. Since it is a random number sampling procedure, it cannot give the theoretical optimum point. However, depending on the size of the sample used and the particular characteristics of the specific problem, the probability of the answer being within a given percent of the theoretical optimum can be made as high as desired. Its advantages are mainly:

1) The answers are direct. The values of the design variables for the optimum design configuration are the output.

2) There are no restrictions on any of the constraint functions nor on the merit function.

3) Constraint functions and the merit function need not be explicitly expressable in terms of the design variables.

4) Any number of variables and constraint conditions can be used, provided the storage capacity of the computer is not exceeded.

5) Integer-valued design variables can be included.

6) It does not require considerable machine time. For example, execution time for a 4-variable, 4-constraint design problem is 2 sec in the IBM 7094 computer.

7) Once the random-sampling main program is established, it can be used for any design problem by changing only the constraint subprograms, since the main constraints are the only variable portion of different synthesis problems.

Its usefulness can be increased by increasing the number of subprograms to cover most of the different types of design problems commonly encountered. Also, the improvement of the execution speed by means of more advanced programming techniques will enable the user to increase the number of independent trials economically, and it will increase its confidence in the dependability of the recommended design configurations.

Appendix A: Application to Symmetric Waffle Plate Problem

There was only one available work of structural synthesis with examples concerning multiple combined load conditions.[4] For that reason, an identical problem with identical constraint conditions was chosen for the purpose of comparison. Five runs were made with different sample sizes and different random number starters in order to obtain an idea about the consistency of the results. The output corresponding to the minimum weight values at the end of the

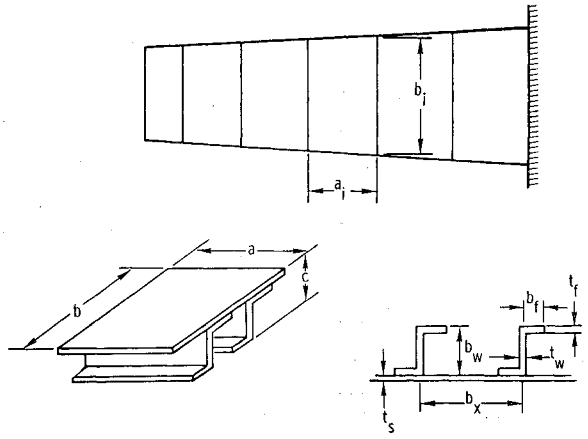


Fig. 5 Stiffened cover plate.

fifth run is tabulated in Table 1, and the Schmit and Kicher output is included for the purpose of comparison.[4]

Appendix B: Application to Stiffened Plates

The application of the random-sampling method to the wing panel design problem consists of defining the variables and the constraint conditions. Several types of stiffeners were investigated for an actual preliminary design problem. However, only a typical Z-stiffened panel is given here for the purpose of demonstration. (See Figs. 5 and 6.) A sample computer output as obtained from a cathode ray tube is also included in Table 3.

For uniaxially stiffened plates subjected to axial compression, only four constraint conditions had to be checked, as follows: 1) Yield condition

$$g_1 = -N_x/(Y.\bar{t}) \tag{B1}$$

2) Buckling of cover sheet between two stiffeners as a simply supported long plate

$$g_2 = \frac{-N_x}{\{[4\Pi^2 E/12(1 - \mu^2)][ts/(b_x - t_w)]\}\bar{t}} \tag{B2}$$

3) Buckling of stiffener web as a long plate simply supported on three sides and free on one side

$$g_3 = \frac{-N_x}{[0.435\ \Pi^2 E/12(1 - \mu^2)(t_w/b_w)^2]\bar{t}} \quad \text{Bleich}^6 \tag{B3}$$

However, in this program an empirical formula was used for g_3:

$$g_3 = \frac{-N_x}{[\text{coeff (Y.E.)}^{1/2}(t_w/b_w)^{3/4}]\bar{t}} \tag{B4}$$

4) Buckling of the panel as an orthotropic plate. For a plate of this type, we can use the Timoshenko[7] equations and, assuming one half-wave buckling pattern, we have

$$N_{cr} = \frac{\Pi^2}{b^2}\left(D_1\frac{b^2}{a^2} + 2D_3 + D_2\frac{a^2}{b^2}\right) \tag{B5}$$

where

$$D_1 = EI_x/(1 - \mu_x\mu_y) \tag{B6}$$

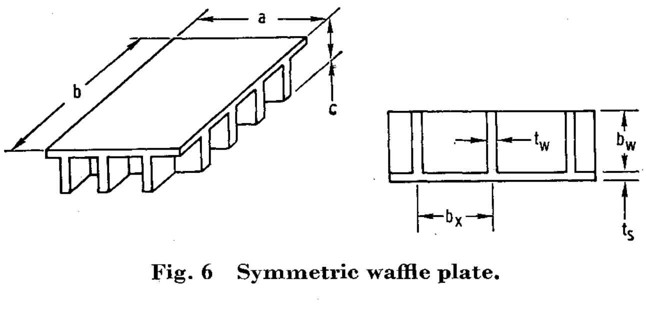


Fig. 6 Symmetric waffle plate.

Table 3 Design configuration by random sampling

Material properties	E = 17500000.	SCY[a] = −125000.				μ = 0.290
	η = 0.00					ρ = 0.160
Type construction—Z-stiffened cover panel						
Load conditions	1	2	3	4	5	6
Nx	−7.00					
Ny	−0.00					
Nxy	−0.00					
Geometry	A = 15.00		S = 110.00		C = 1.20	
Constant values of variables						
VIN =	0.0000	−0.0000	−0.0000	2.6900	0.7710	−0.0000
Min-max values of variables						
$VMIN$ =	0.0160	0.8000	0.0160			
$VMAX$ =	0.1000	1.2000	0.1000			
Optimum values of design variables						
Approximation 1						
V =	0.082	0.920	0.072			
	weight =	32.65(lb)	TBAR[b] = 0.1237			
G = 0.453, 0.923, 0.335, 0.394 . . .						
Approximation 2						
V =	0.089	0.908	0.034			
	weight =	28.58 (lb)	TBAR = 0.1083			
G = 0.517, 0.921, 0.666, 0.462 . . .						
Approximation 3						
V =	0.089	0.906	0.023			
	weight =	26.95(lb)	TBAR = 0.1021			
G = 0.549, 0.985, 0.937, 0.491 . . .						
Approximation 4						
V =	0.089	0.904	0.023			
	weight =	26.92 (lb)	TBAR = 0.1020			
G = 0.549, 0.984, 0.945, 0.496 . . .						

[a] SCY = yield stress.
[b] TBAR = average thickness.

$$D_2 = EI_s/(1 - \mu_x\mu_y) \tag{B7}$$

$$D_3 = \tfrac{1}{2}(\mu_x D_2 + \mu_y D_1) + 2GI_{xy} \tag{B8}$$

For isotropic plates $D_1 = D_2 = D_3$ and for orthotropic plates, we can utilize the conservative assumptions that $D_3 = D_2$, and $\mu_y = \mu_x$, which give

$$g_4 = \frac{-N_x}{[\Pi^2 E/(1 - \mu^2)](1/a^2)\{I_x + I_s[2(a^2/b^2) + (a^4/b^4)]\}} \tag{B9}$$

The results of the optimization program are tabulated in Table 2 and the input information is as follows:

For Z-stiffeners:

$$V(1) = t_s \qquad V(2) = b_w \qquad V(3) = t_w$$

$$b_f = 0.35\ b_w \qquad t_f = t_w \qquad C = 1.500$$

$$Y = 65 \text{ ksi} \qquad \rho = 0.10 \text{ lb/in.}^3$$

$$E = 10^4 \text{ ksi} \qquad \mu = 0.32$$

$$0.350 \leqslant b_w \leqslant C - t_s \qquad 0.020 \leqslant t_w \leqslant 0.250$$

The merit function for these examples is the weight of the panel. $F = abt\rho$.

References

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